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The research group studied the following problems in the field of the theory of hydrodynamics: 1. Integrability of free surface hydrodynamics and five-wave interaction for gravity waves; 2. Singularity formation on the free surface of an ideal fluid; 3. Wave dynamics and collapse in shear-flows; 4. Turbulent Kolmogorov spectra for capillary waves.

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"Air-Sea Exchanges of Energy and Momentum under Well-Developed Sea Conditions: Theory and Experiment"

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Final Report

The research group studied the following problems in the field of the theory of the hydrodynamics. Results of the collaboration by Dr. R. E. Glazman at the Jet Propulsion Laboratory are found in Appendix I.

1. Integrability of Free Surface Hydrodynamics and Five-Wave Interaction for Gravity Waves

It is shown in (1) that the set of equation for the gravity wave propagation on the surface of the fluid of infinite depth is intergable one up to the fourth order in the perturbation series in the powers of complex normal variables. Based on the Hamiltonian approach, we have shown that the scattering matrix for four-wave interaction of the gravity waves is identically equal to zero for the one-dimensional case. We have also made a great deal of progress in understanding the importance of the five-wave interaction for gravity wave turbulence. The elegant form of the scattering matrix for that interaction is calculated for the one-dimensional case (2).

As a side effect of that activity we have shown that two approaches for statistical description of gravity wave turbulence introduced by K.Hasselmann and V.Zakharov result in the same kinetic equation for the second order correlator (3).

2. Singularity Formation On the Free Surface of an Ideal Fluid

The method of conformal mapping is used for the description of the nonlinear dynamics of an ideal fluid with a free surface. Unlike the commonly-used Birkhoff-Rott equations, based on the Lagrangian approach, the equations obtained are pseudo-differential ones, containing only the Hilbert transform on the straight line only. These (exact) equations are much simpler and better adjusted for numerical simulations (4). The numerical code was also developed.

We have shown that the equations of motion of an ideal fluid with a free surface in the absence of both gravitational and capillary forces can be effectively solved in the approximation of small surface angles (5). It is done by analytical continuation for both the velocity potential of the surface and its elevation. For most initial conditions, the system evolves to form singularities in a finite time. The types of different singularities are classified. All of them correspond to the moving branch points in the complex plane, and singularity appeas when such a branch point hits the real axis which corresponds to the physical space.

3. Wave Dynamics and Collapse in Shear-Flows

The sheared variables, corresponding to the amplitudes of the convective modes, are shown (6) to diagonalize the quadratic Hamiltonian, i.e., to be normal variables of the Hamiltonian equations while the convective modes are the normal ones in the Hamiltonian dynamics as the basis for the nonlinear theory.

In the framework of the generalized two-dimensional Benjamin-Ono equation, we have studied both analytically and numerically the nonlinear evolution of "quasi" one-dimensional perturbations of the shear flow. We have shown that in the limit of small viscosity the instability of the one-dimensional soliton,

perturbed in the transverse direction, results in the self-focusing and collapse. It leads to the separation of a one-dimensional soliton into individual clusters and to the subsequent self-focussing of each cluster. With the help of the embedding inequalities, an upper boundary for the noncollapsing sector is found. Both the theoretical and numerical results obtained are in a qualitative agreement with experimental data on coherent structures in the boundary layer.

4. Turbulent Kolmogorov Spectra for Capillary Waves

The behavior of an ensemble of capillary waves on the surface of incompressible fluid of infinite depth was studied numerically (8). Driving force (external pressure) localized at small wavenumbers and an energy absorbtion was provided by the viscosity dumping acting at high wavenumbers. It is found that the picture of turbulence depends dramatically on the nature of long-wave driving force, i.e., whether the driving force is in or out of resonance with local linear frequency of the system. Resonant or non-resonant pumping defines correspondingly two different regimes of turbulence: first, one without energy flux to the damping region of small scales (interpreted as KAM regime) and second, one with the finite energy flux. In the latter, the formation of power-like Kolmogorov spectra was observed. Index of this spectra corresponds to that predicted by "weak turbulence" approach.

For the mean field approximation for the Euler equation the Kolmogorov constant for constant energy flux was calculated (9). The results of (1), (4), (5), and (8) were reported at ONR "Ocean Waves" Workshop, 1994, Tucson. (Appendix II). The results of (1), (2), (4), (5) and (7) were reported at "New achievments in the Nonlinear Schroedinger Equation"-Workshop, Landau Institute for Theoretical Physics, Moscow, Russia, 1994.

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APPENDIX I

Subcontract to ONR Contract No. N00014-92-J-1343

R. E. Glazman
Jet Propulsion Laboratory
Pasadena, CA

I. Theory

1. We now understand the fundamental role of surface tension in explaining the presence of two regimes of a wind-driven sea surface. For the wind speed U less than a critical value [1], [2], [9],

$$U_0 \sim \left(\frac{\rho_w}{\rho_a}\right)^{1/2} (\sigma g)^{1/4}, \tag{1}$$

the surface tension dominated wrinkling of the surface can absorb the direct energy cascade and the surface stays smooth. For $U > U_0$, however, the Kolmogorov equilibrium spectrum

$$E_1(k) \sim k_0^4 \langle \eta_k^4 \rangle = \left(\frac{P}{P_0}\right)^{1/3} \left(\frac{k_0}{k}\right)^{7/2} \tag{2}$$

corresponding to a constant energy flux $P\left(P = \left(\frac{\rho_a}{\rho_w}\right)^{3/2} U^3, k_0 = (g/\sigma)^{1/2}\right)$, meets the Phillips spectrum

$$E_3(k) = \left(\frac{k_0}{k}\right)^4 \tag{3}$$

at a value $k_1 < k_0$, that is at wavelengths much larger than those at which surface tension effects come into play. The result is that the surface develops singular derivatives and in order to create sufficient surface area to absorb the energy flux, the surface breaks and issues forth a cloud of water droplets or foam. A first attempt to describe some of the characteristics of this new phase (foam height, water droplet size) in terms of the parameters $P, P_0 = \left(\frac{\rho_w}{\rho_a}\right)^{3/2} (\sigma g)^{3/4}, k_0$ is given in [2].

2. If surface tension effects and three wave interactions can be neglected, then the first closure of the moment equations at order ε^4 gives rise to the well known Hasselmann kinetic equation for a system of random dispersive waves which share energy via four wave resonances. It is important to study also the situation at the next order, for two reasons. First, over longer times than ε^{-4}, the five wave resonances are important. Second, and equally interesting, is the fact that at this order, the wave action $\int n_k d\vec{k}$ is no longer a motion constant. This can have important ramifications for the inverse cascade and the understanding of the aging process, namely the presence of waves with phase speeds greater than the wind speed. Using Feynmann diagram techniques, this kinetic equation has been derived by V. P. Krazitzky and V. E. Zakharov [3], [4]. Weak turbulent Kolmogorov-like spectra for multiwave interactions have been found recently by Glazman [6].

3. We have made some progress towards understanding the nature of wave breaking. We consider that the wave breaking results from the spontaneous formation of angle-type singularities of the free fluid surface. A self-similar theory of such singularities was developed in the paper [1] and a more advanced theory is contained in [10].

II. Analysis of in situ data

This analysis was performed by R. Glazman and published in the articles [5, 7-8]. The analysis includes measurements of wave spectra in the open sea by buoy-installed devices, studying of duration and fetch dependence of the characteristic wavelength and wave energy. The results are compared with the predictions of the weak turbulent theory. Measurements have quantitatively confirmed the existence of direct and inverse cascades of energy. The shape of the spectrum in the direct cascade area is very well approximated by the weak turbulence formula $I(\omega) \simeq \omega^{-4}$. In the inverse cascade area $\left(\omega \simeq \frac{g}{u}\right)$ measurements give $I(\omega) \simeq \omega^{-s}$ and s is 3. The weak-turbulence prediction 11/3. The fetch dependence of the characteristic wavelength is power-like

$$S = Cx^c$$

and c varies from 0.29 to 0.33. The weak turbulence theory predicts $c = \frac{3}{19} \simeq 0.22$. Sizable deviations from the weak turbulence theory can perhaps

be explained by the facts that long wave damping due to friction with the turbulent air is neglected and that the interaction with the shear flow near the surface is ignored.

III. Plans for the next stage

1. It is now clear that a whole new theory of surface wave theory must be developed in which the sea surface is no longer a discontinuity dividing air and water but consists of a finitely thick layer of a new phase, a foam mixture of air and water. Our first attempts to describe some of the characteristics of this phase in [2] were very elementary and what is needed is a new dynamical theory which takes account of water droplets in air, air bubbles contained in water (here compressibility may be important). We must also include nonpotential flow effects as the momentum absorbed by the water layer from the waves will create a surface shear flow.

2. We plan to develop a numerical simulation of the existing water wave equations (with nonbroken surface) much in the same vein as we studied optical turbulence by integrating the nonlinear Schrödinger equation [11]. The results should be compatible with the results of simulations of the kinetic equation but will allow us also to understand the transition regimes between waves dominated by gravity and those dominated by surface tension. The key step in this work is to formulate the water wave equations as a <u>local</u> evolution of some appropriately defined variables of x and y, the surface coordinates. We have some good ideas. This part of the work will be carried out by Drs. S. Dyachenko and A. Pushkarev who will be at Arizona from July 1993 to August 1994.

3. We shall also be studying models for explaining the observed angular distribution of energy in long waves. The distribution is more directed along the wind direction than would be predicted by present theories.

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APPENDIX II

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